Galois Theory Mid Term

This exam is of **30 marks** and is **3 hours long**. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly.

1. Let V be a three dimensional vector space over Q. If $v = (a_1, b_1, c_1)$ and $w = (a_2, b_2, c_2)$ define

$$v * w = (a_1a_2 + 2b_1c_2 + 2c_1b_2, a_1b_2 + b_1a_2 + 2c_1c_2, a_1c_2 + b_1b_2 + c_1a_2)$$

(4)

Show that (V, *, +) is a field.

2. Answer **True** or **False**. If **True**, prove it, if **False**, give a counterexample. Let $F \subseteq L \subseteq K$ be fields.

1. If L/F and K/L are normal , then K/F is normal .	(3)
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- 2. If L/F and K/L are separable, then K/F is separable. (3)
- 3. If L/F and K/L are purely inseparable, then K/F is purely inseparable. (3)
- 4. If K/L is normal and L/F is purely inseparable then K/F is normal. (3)
- 5. If K/F is purely inseparable then K/L and L/F are purely inseparable. (3)
- 6. If K/F is separable then K/L and L/F are separable. (3)
- 7. If K/F is **normal** then K/L and L/F are **normal**. (3)
- 8. If K/F is normal then K/L is normal and L/F is purely inseparable. (3)
- 3. Give an example of a field that is neither **normal** nor **separable**. (2)